Problem Set on STR-1

1. Apply the Principle of Relativity to Newton's 2nd Law to prove that any two inertial frames are related by a Galilean transformation

$x \rightarrow x' = Rx + c + ut$ where R, c and u are constant.

Derive therefrom the Galilean velocity addition theorem: $v' = v + u$.

2. Discuss invariance of Newton's first and second laws with respect to Galilean time translation and scaling: $t \rightarrow t' = \alpha t + \beta$ (where α and β are constant)

3. Inside a cable car climbing a slope with constant velocity v0 an object is dropped from rest. Derive equations for the trajectory within the car and with respect to the earth outside.

4. Use Galileo's Velocity addition rule. Let S be an inertial reference system.

(a) Suppose that S' moves with constant velocity relative to S. Show that S' is also an inertial reference system.

(b) Conversely, show that if S' is an inertial reference system, then it moves with respect to S at constant velocity.

5. As an illustration of the principle of relativity in classical mechanics, consider the following generic conditions: In initial frame S, particle A (mass m_A, velocity $\overrightarrow{u_A}$) hits particle B (mass m_B, velocity $\overrightarrow{u_B}$). In the course of the collision some mass rubs off A and onto B, and we left with particle C (mass m_C, velocity $\overrightarrow{u_C}$) and D (mass m_D, velocity $\overrightarrow{u_D}$). Assume that momentum is conserved in S.

(a) Prove that momentum is also conserved in inertial frame S', which moves with velocity $\vec{v}$ relative to S.

(b) Suppose the collision is elastic in S, show that it is also elastic in S'.

6. As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. According to Galileo, does the bullet reach its target?